



Lesson 16: Relating Scale Drawings to Ratios and Rates

Student Outcomes

- Students understand that a scale drawing is either the reduction or the enlargement of a two-dimensional picture.
- Students compare the scale drawing picture with the original picture and determine if the scale drawing is a reduction or an enlargement.
- Students match points and figures in one picture with points and figures in the other picture.

Classwork

Opening Exercise (3 minutes): Can You Guess the Image?

Project the Opening Exercise pages at the end of the lesson to show students a series of images. Determine if they can guess what is pictured and then identify whether the picture is a reduction or an enlargement of the original image. The purpose of this activity is for students to understand the terms *reduction* and *enlargement*. The scale drawings produced in Grade 7 focus on creating a scale drawing from a two-dimensional picture. Teachers can also post alternate images of choice on a projector or interactive whiteboard where only one portion is revealed. Students guess the object and determine if the picture is a reduction or an enlargement of the actual object.

Scaffolding:

In each Opening Exercise problem, the image can be resized on a digital display for students to see the enlarged version of the original picture.

Give students two minutes to guess each image in the student materials and share responses. Then show the full-size images, and have students decide whether the images in the student materials are reductions or enlargements, compared to what is now being shown.

Responses for attached images and points for discussion follow.

- This is a picture of a subway map. Was the cropped photo that was just seen a reduction or an enlargement of the original picture below? How do you know?
 - It is a reduction since it is a scaled down picture of a map of a subway. If you compare the length from one end of a track to the other end, you can see that the cropped photo has a shorter length as compared to the original photo. Any corresponding length could be compared.*
- This is a fingerprint. Was the cropped photo that was just seen a reduction or an enlargement of the original picture below? How do you know?
 - It is an enlargement since it was from a picture of a fingerprint. If you compare the length of one of the swirls to the actual fingerprint picture, you can see that the cropped photo has a longer length compared to the original fingerprint picture.*

Opening Exercise: Can You Guess the Image?

1.



This is a reduction of a subway map.

2.



This is an enlarged picture of a fingerprint.

Example 1 (3 minutes)

For each scale drawing, have students identify if it is a reduction or an enlargement of the actual object in real life or of the given original picture.

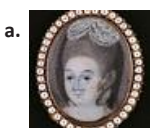
- What are possible uses for enlarged drawings/pictures?
 - *Enlarged drawings allow us to observe details such as textures and parts that are hard to see to the naked eye. In art, enlargements are used in murals or portraits.*
- What are the possible purposes of reduced drawings/pictures?
 - *Reductions allow us to get a general idea of a picture/object. These scale drawings can fit in folders, books, wallets, etc.*

Introduce the term *scale drawing*. Emphasize the importance of scale drawings being reductions or enlargements of two-dimensional drawings, not of actual objects.

Example 1

For the following problems, (a) is the actual picture, and (b) is the scale drawing. Is the scale drawing an enlargement or a reduction of the actual picture?

1.



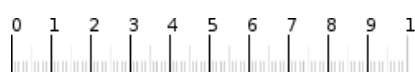
b.



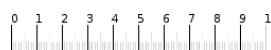
Enlargement

2.

a.



b.



Reduction

SCALE DRAWING: A reduced or enlarged two-dimensional drawing of an original two-dimensional drawing.

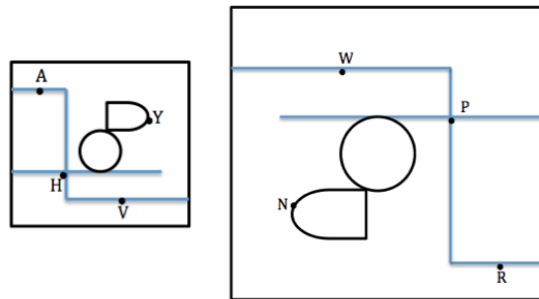
Example 2 (7 minutes)

Complete this activity together as a class.

- Why doesn't point V correspond with point R ?
 - *Although both points are on the bottom right hand corner, they are positioned on two different ends of the path. Point V only corresponds to Point W .*
- What must we consider before identifying corresponding points?
 - *We have to make sure we are looking at the maps in the same direction. Then we can see that this is a one-to-one correspondence because they are scale drawings of each other, and each point corresponds to one specific point on the other map.*

Example 2

Derek's family took a day trip to a modern public garden. Derek looked at his map of the park that was a reduction of the map located at the garden entrance. The dots represent the placement of rare plants. The diagram below is the top-view as Derek held his map while looking at the posted map.



What are the corresponding points of the scale drawings of the maps?

Point A to Point R Point V to Point W Point H to Point P Point Y to Point N

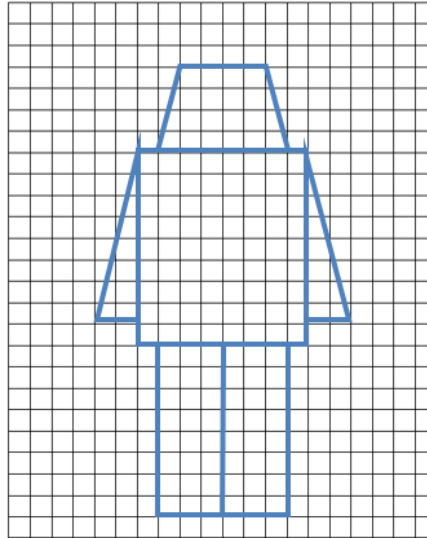
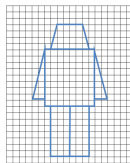
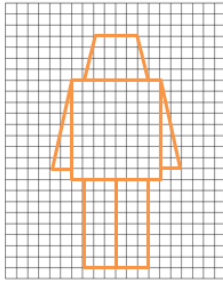
Exploratory Challenge (10 minutes)

In this exercise, the sizes of the units on the grid are enlarged, and then reduced to produce two different scale drawings with lengths that are proportional to one another. Guide students to notice that the number of units of length is staying the same, but the size of each unit changes from one drawing to another due to the shrinking and enlarging of the grid. This allows for students to create a scale drawing without having to complete the computation required in finding equivalent ratios (which is done later in Topic D).

- How will we make the enlarged robot? Will we need to adjust the number of units?
 - *No, since the grid is enlarged (thus changing the size of each unit), we can draw the new robot using the same number of units for each given length.*
- What is the importance of matching corresponding points and figures from the actual picture to the scale drawing?
 - *If there is no correspondence between points from the actual picture to the scale drawing, the scale drawing will not be proportional, and the picture will be distorted.*
- How can you check the accuracy of the proportions?
 - *You can count the squares and check that the points match.*

Exploratory Challenge

Create scale drawings of your own modern nesting robots using the grids provided.

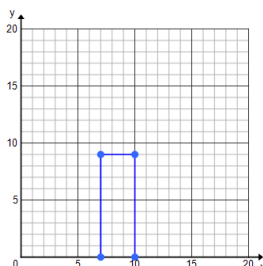
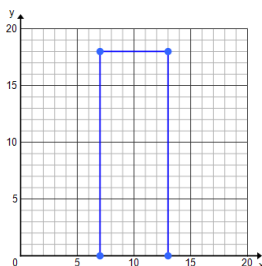
**Example 3 (7 minutes)**

Work on the problem as a class and fill in the table together. Discuss as students record important points in the “Notes” section:

- Is the second image a reduction or an enlargement of the first image? How do you know?
 - *It is a reduction because the second image is smaller than the first original image.*
- What do you notice about the information on the table?
 - *The pairs of corresponding lengths are all proportional.*
- Does a constant of proportionality exist? How do you know?
 - *Yes, it does because there is a constant value to get from each length to its corresponding length.*
- What is the constant of proportionality, and why is it important in scale drawings?
 - *The constant of proportionality is $\frac{1}{2}$, and it needs to exist for images to be considered scale drawings. If not, then there would be a lack of proportionality, and the images would not be a scaled up or down version of the original image.*

Example 3

Celeste drew an outline of a building for a diagram she was making and then drew a second one mimicking her original drawing. State the coordinates of the vertices and fill in the table.



	Height	Length
Original Drawing	18 units	6 units
Second Drawing	9 units	3 units

Notes:

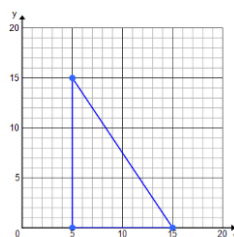
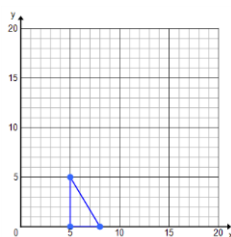
Exercise (7 minutes)

Have students work in pairs to fill out the table and answer the questions. Direct students to the vertical and horizontal lengths of the legs. Reconvene as a class to discuss answers to the given questions and the following:

- Why is it difficult to determine if the second image is a reduction or an enlargement of the first image?
 - *The second image is not a scale drawing of the first image. Although the second image is bigger, it is not a true reduction or enlargement of the first image.*
- What must one check before determining if one image is a scale drawing of another?
 - *The corresponding lengths must all be proportional to each other. If only one pair is proportional and another is not, then the images cannot be identified as scale drawings of one another.*

Exercise

Luca drew and cut out a small right triangle for a mosaic piece he was creating for art class. His mother liked the mosaic piece and asked if he could create a larger one for their living room. Luca made a second template for his triangle pieces.



	Height	Width
Original Image	5 units	3 units
Second Image	15 units	10 units

- a. Does a constant of proportionality exist? If so, what is it? If not, explain.
No, because the ratios of corresponding side lengths are not equal or proportional to each other.
- b. Is Luca's enlarged mosaic a scale drawing of the first image? Explain why or why not.
No, the enlarged mosaic image is not a scale drawing of the first image. We know this because the images do not have all side lengths proportional to each other; there is no constant of proportionality.

Closing (3 minutes)

- What is a scale drawing?
 - *It is a drawing that is a reduction or an enlargement of an actual picture.*
- What is an enlargement? What is a reduction?
 - *An enlargement is a drawing that is larger in scale than its original picture. A reduction is a drawing that is smaller in scale than its original picture.*
- What is the importance of matching points and figures from one picture or drawing to the next?
 - *The corresponding lines, points, and figures need to match because otherwise the scale drawing will be distorted and not proportional throughout.*
- How do scale drawings relate to rates and ratios?
 - *The corresponding lengths between scale drawings and original images are equivalent ratios.*

Lesson Summary

SCALE DRAWING AND SCALE FACTOR (description): For two figures in the plane, S and S' , S' is said to be a *scale drawing* of S with *scale factor* r if there is a one-to-one correspondence between S and S' so that, under the pairing of this one-to-one correspondence, the distance $|PQ|$ between any two points P and Q of S is related to the distance $|P'Q'|$ between corresponding points P' and Q' of S' by $|P'Q'| = r|PQ|$.

A scale drawing is an *enlargement* or *magnification* of another figure if the scale drawing is larger than the original drawing, that is, if $r > 1$.

A scale drawing is a *reduction* of another figure if the scale drawing is smaller than the original drawing, that is, if $0 < r < 1$.

Exit Ticket (5 minutes)

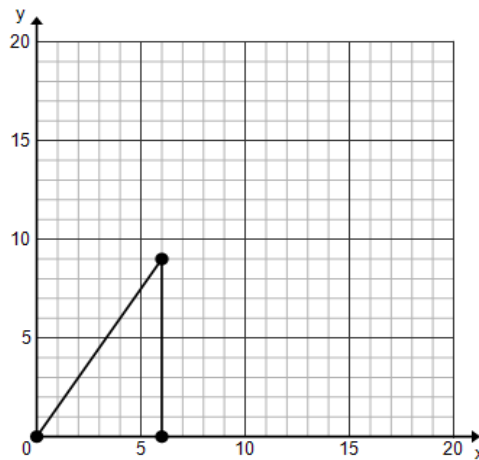
Name _____

Date _____

Lesson 16: Relating Scale Drawings to Ratios and Rates

Exit Ticket

Use the following figure on the graph for Problems 1 and 2.



- 1.
- If the original lengths are multiplied by 2, what are the new coordinates?

- Use the table to organize lengths (the vertical and horizontal legs).

	Width	Height
Actual Picture (in units)		
New Picture (in units)		

- Is the new picture a reduction or an enlargement?
- What is the constant of proportionality?

2.

a. If the original lengths are multiplied by $\frac{1}{3}$, what are the new coordinates?

b. Use the table to organize lengths (the vertical and horizontal legs).

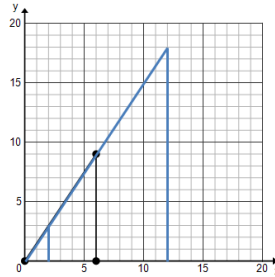
	Width	Height
Actual Picture (in units)		
New Picture (in units)		

c. Is the new picture a reduction or an enlargement?

d. What is the constant of proportionality?

Exit Ticket Sample Solutions

Use the following figure on the graph for Problems 1 and 2.



1.

- a. If the original lengths are multiplied by 2, what are the new coordinates?

$(0, 0)$, $(12, 18)$, $(12, 0)$

- b. Use the table to organize lengths (the vertical and horizontal legs).

	Width	Height
Actual Picture (in units)	6 units	9 units
New Picture (in units)	12 units	18 units

- c. Is the new drawing a reduction or an enlargement?

Enlargement

- d. What is the constant of proportionality?

2

2.

- a. If the original lengths are multiplied by $\frac{1}{3}$, what are the new coordinates?

$(0, 0)$, $(2, 3)$, $(2, 0)$

- b. Use the table to organize lengths (the vertical and horizontal legs).

	Width	Height
Actual Picture (in units)	6 units	9 units
New Picture (in units)	2 units	3 units

- c. Is the new drawing a reduction or an enlargement?

Reduction

- d. What is the constant of proportionality?

$\frac{1}{3}$

Problem Set Sample Solutions

For Problems 1–3, identify if the scale drawing is a reduction or an enlargement of the actual picture.

1. Enlargement

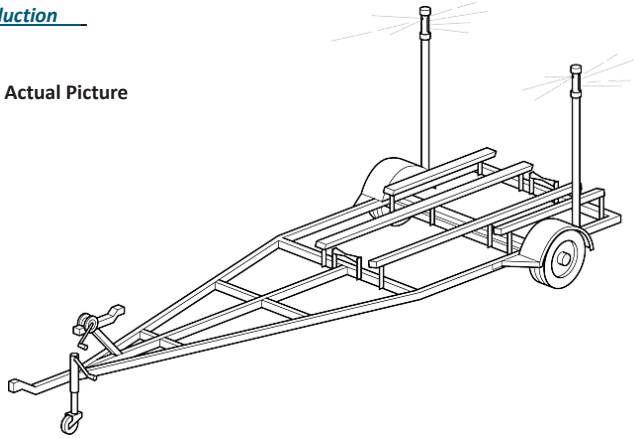
a. Actual Picture



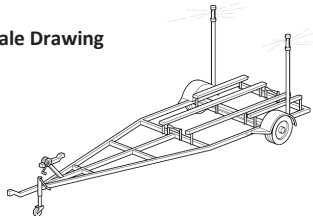
b. Scale Drawing

2. Reduction

a. Actual Picture



b. Scale Drawing

3. Enlargement

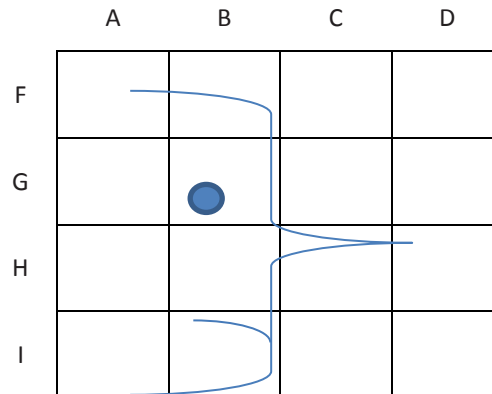
a. Actual Picture



b. Scale Drawing

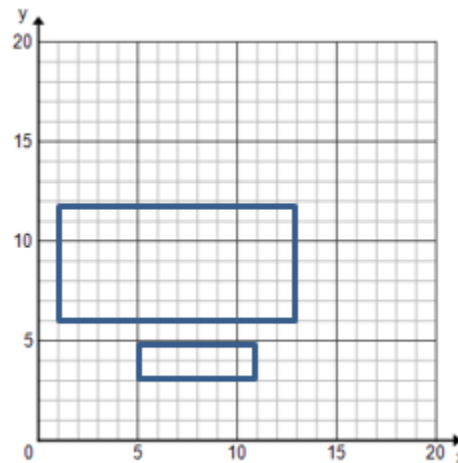


4. Using the grid and the abstract picture of a face, answer the following questions:



- On the grid, where is the eye?
Intersection BG
 - What is located in DH ?
Tip of the nose
 - In what part of the square BI is the chin located?
Bottom right corner
5. Use the blank graph provided to plot the points and decide if the rectangular cakes are scale drawings of each other.
- Cake 1: $(5, 3), (5, 5), (11, 3), (11, 5)$
- Cake 2: $(1, 6), (1, 12), (13, 12), (13, 6)$
- How do you know?

These images are not scale drawings of each other because the short length of Cake 2 is three times longer than Cake 1, but the longer length of Cake 2 is only twice as long as Cake 1. Both should either be twice as long or three times as long to have one-to-one correspondence and to be scale drawings of each other.



Opening Exercise

Can you guess the image? In each problem, the first image is from the student materials, and the second image is the actual picture.

1.



2.

