

# Learning Standards for High School

\*Additional mathematics standards for High School that represent complex numbers on the complex plane in rectangular and polar form (including real and Imaginary numbers) and are indicated by a + symbol in the Ohio's New Learning Standards are not included in the extended standards since they are not considered common mathematics curriculum for all college and career ready students.


## NUMBERS AND QUANTITY


Learning Standard	Complexity a	Complexity b	Complexity c
Most Complex	Least Complex		
The Real Number System			
Extend the properties of exponents to rational exponents.			
N.RN.1 Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents. For example, we define $5^{1/3}$ to be the cube root of 5 because we want $(5^{1/3})^3 = 5^{(1/3)3}$ to hold, so $(5^{1/3})^3$ must equal 5.	N.RN.1a Identify equivalent expression with exponents (e.g. $3 \times 3 \times 3 \times 3$ is $3^4$ ).	N.RN.1b Identify equivalent expression with exponents (limit to squares) (e.g. $3 \times 3$ is $3^2$ ).	N.RN.1c Identify a number with an exponent.
N.RN.2 Rewrite expressions involving radicals and rational exponents using the properties of exponents.	N.RN.2a Identify equivalent expression with exponents and radicals (e.g. $3 \times 3 \times 3 \times 3$ is $3^4$ ).	N.RN.2b Identify equivalent expression with exponents and radicals (e.g. $3 \times 3$ is $3^2$ ).	N.RN.2c Identify a number with a radical.
Use properties of rational and irrational numbers.			
N.RN.3 Explain why the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational.	N.RN.3a Recognize the effects of multiplying and dividing with negative numbers (e.g., $-2 \times -4 = 8$ ).	N.RN.3b Recognize that the absolute value of a rational number is how far it is from 0 on the number line (e.g., plot a number and its opposite on a number line and recognize that they are equidistant from zero).	N.RN.3c Recognize that addition means move to the right and subtraction means move to the left on a number line.

Learning Standard	Complexity a	Complexity b	Complexity c
Most Complex	←	→	Least Complex
<b>Quantities</b>			
<i>Reason quantitatively and use units to solve problems.</i>			
<b>N.Q.1</b> Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.	<b>N.Q.1a</b> Solve real-world problems involving positive and negative numbers (e.g., temperatures, elevations, and distance from a fixed point (map reading)).	<b>N.Q.1b</b> Solve problems involving positive and negative numbers using a number line (e.g., temperatures, distances from a fixed point).	<b>N.Q.1c</b> Locate a given positive or negative number on a number line.
<b>N.Q.2</b> Define appropriate quantities for the purpose of descriptive modeling.	<b>N.Q.2a</b> Identify the appropriate unit of measure for volume.	<b>N.Q.2b</b> Identify the appropriate unit of measure for length.	<b>N.Q.2c</b> Identify units of measure.
<b>N.Q.3</b> Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.	<b>N.Q.3a</b> Identify the appropriate unit of measure for volume.	<b>N.Q.3b</b> Identify the appropriate unit of measure for length.	<b>N.Q.3c</b> Identify units of measure.
<b>The Complex Number System</b>			
<i>Perform arithmetic operations with complex numbers.</i>			
<b>N.CN.1</b> Know there is a complex number $i$ such that $i^2 = -1$ , and every complex number has the form $a+bi$ with $a$ and $b$ real.	<b>N.CN.1a</b> Describe real and complex numbers.	<b>N.CN.1b</b> Given a set of numbers, compare the real and complex numbers.	<b>N.CN.1c</b> Given a set of numbers, label the real and complex numbers.
<b>N.CN.2</b> Use the relation $i^2 = -1$ and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers.	<b>N.CN.2a</b> Add and subtract complex numbers.	<b>N.CN.2b</b> Identify a complex number.	<b>N.CN.2c</b> Identify an imaginary number.
<i>Use complex numbers in polynomial identities and equations.</i>			
<b>N.CN.7</b> Solve quadratic equations with real coefficients that have complex solutions.	<b>N.CN.7a</b> Solve a quadratic equation with real solutions.	<b>N.CN.7b</b> Given a graph of a quadratic equation, match the correct equation.	<b>N.CN.7c</b> Identify a coefficient.

## ALGEBRA

Most Complex		Least Complex	
Interpret the structure of expressions.			
<p><b>A.SSE.1a</b> Represent a real-world situation with an expression, both numerals and variables. Recognize parts of the expression in the real-world situation.</p>	<p><b>A.SSE.1b</b> Represent a real-world situation with a numeric expression. Recognize parts of the expression in the real-world situation.</p>	<p><b>A.SSE.1c</b> Represent a real-world situation with a model using concrete objects.</p>	
<p><b>A.SSE.2a</b> Simplify expressions involving variables (e.g., <math>(2(x + 4) = 2x + 8)</math>).</p>	<p><b>A.SSE.2b</b> Identify the equivalent numeric expression (e.g., <math>7 + 5 = 5 + 7</math>).</p>	<p><b>A.SSE.2c</b> Identify equivalent expressions with whole numbers less than 10 using concrete objects (e.g., objects, dots, etc.).</p>	
Write expressions in equivalent forms to solve problems.			
<p><b>A.SSE.3</b> Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.</p> <p><b>a.</b> Factor a quadratic expression to reveal the zeros of the function it defines. (A1, M2)</p> <p><b>b.</b> Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines. (A1, M2)</p> <p><b>c.</b> Use the properties of exponents to transform expressions for exponential functions. For example, <math>8^t</math> can be written as <math>2^{3t}</math>.</p>	<p><b>A.SSE.3a</b> Apply properties of integer exponents to generate equivalent variable expressions (e.g., <math>b^2 \times b^4 = b^6</math>).</p>	<p><b>A.SSE.3b</b> Apply properties of integer exponents to generate equivalent numerical expressions (e.g., <math>5^2 \times 5^4 = 5^6</math>).</p>	<p><b>A.SSE.3c</b> Interpret numerical expressions with exponents (e.g., <math>5^4</math> means <math>5 \times 5 \times 5 \times 5</math>).</p>

Learning Standard		Complexity a	Complexity b	Complexity c
Most Complex				Least Complex
Arithmetic with Polynomials and Rational Expression Standards				
Perform arithmetic operations on polynomials.				
<b>A.APR.1</b> Understand that polynomials form a system analogous to the integers, namely, that they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials. <b>a.</b> Focus on polynomial expressions that simplify to forms that are linear or quadratic. (A1, M2) <b>b.</b> Extend to polynomial expressions beyond those expressions that simplify to forms that are linear or quadratic. (A2, M3)	<b>A.APR.1a</b> Add and subtract linear and/or quadratic polynomials. Models may be used.	<b>A.APR.1b</b> Add and subtract linear polynomials. Models may be used.	<b>A.APR.1c</b> Add linear polynomials. Models may be used.	
Understand the relationship between zeros and factors of polynomials.				
<b>A.APR.2</b> Understand and apply the Remainder Theorem: For a polynomial $p(x)$ and a number $a$ , the remainder on division by $x - a$ is $p(a)$ . In particular, $p(a) = 0$ if and only if $(x - a)$ is a factor of $p(x)$ .	<b>A.APR.2a</b> Multiply two binomials.	<b>A.APR.2b</b> Multiply a variable by a binomial.	<b>A.APR.2c</b> Identify a polynomial (binomials only).	
<b>A.APR.3</b> Identify zeros of polynomials, when factoring is reasonable, and use the zeros to construct a rough graph of the function defined by the polynomial.	<b>A.APR.3a</b> Find the zeros of a polynomial when the polynomial is factored (e.g., $x^2 - 9 = 0$ and $x^2 = 3x = 2 = 0$ ).	<b>A.APR.3b</b> Identify a polynomial (trinomial) (e.g., $x^2 = 3x = 2 = 0$ ).	<b>A.APR.3c</b> Identify a polynomial (binomial) (e.g., $x^2 - 9 = 0$ ).	
Rewrite rational expressions.				
<b>A.APR.6</b> Rewrite simple rational expressions in different forms; write $\frac{a(x)}{b(x)}$ in the form $q(x) + \frac{r(x)}{b(x)}$ , where $a(x)$ , $b(x)$ , $q(x)$ , and $r(x)$ are polynomials with the degree of $r(x)$ less than the degree of $b(x)$ , using inspection, long division, or, for the more complicated examples, a computer algebra system.	<b>A.APR.6a</b> Identify a rational expression (e.g., $\frac{6x}{3} = 2x$ ).	<b>A.APR.6b</b> Rewrite expressions in different forms (e.g., $x^2 + 1 = (x * x) + 1$ ).	<b>A.APR.6c</b> Given a visual model, identify an expression (e.g. $2 * 2 = 2^3$ ).	

Learning Standard		Complexity a	Complexity b	Complexity c
Most Complex		Least Complex		
Creating Equations Standards				
Create equations that describe numbers or relationships.				
<b>A.CED.1</b> Create equations and inequalities in one variable and use them to solve problems. Include equations and inequalities arising from linear, quadratic, simple rational, and exponential functions. <b>a.</b> Focus on applying linear and simple exponential expressions. (A1, M1) <b>b.</b> Focus on applying simple quadratic expressions. (A1, M2) <b>c.</b> Extend to include more complicated function situations with the option to solve with technology. (A2, M3)	<b>A.CED.1a</b> Represent and solve a real-world situation with a two-step linear equation or inequality.	<b>A.CED.1b</b> Represent and solve a real-world problem with a one-step linear equation or inequality (e.g., Abby has \$5, and she wants to buy a T-shirt for \$8. How much more money does she need? Key: $5 + x = 8$ ).	<b>A.CED.1c</b> Represent a real-world problem with a linear equation, using concrete objects, models and pictures (see example below). 	
<b>A.CED.2</b> Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales. <b>a.</b> Focus on applying linear and simple exponential expressions. (A1, M1) <b>b.</b> Focus on applying simple quadratic expressions. (A1, M2) <b>c.</b> Extend to include more complicated function situations with the option to graph with technology. (A2, M3)	<b>A.CED.2a</b> Create an equation with two variables to represent a linear relationship between quantities in a given context (e.g., $y = 2x + 4$ ).	<b>A.CED.2b</b> Using a two-variable equation describing a real-world situation, given the value of one variable, find and interpret the value of the other variable (e.g., Sally starts with \$4 and gets an allowance of \$2 each week. After x weeks, she has $y = 2x + 4$ dollars. When $x = 3$ , find y and interpret the result).	<b>A.CED.2c</b> Identify the meaning of each number and/or variable in a given two-variable equation that describe a real-world situation (e.g., Sally starts with \$4 and gets an allowance of \$2 each week. After x weeks she has $y = 2x + 4$ dollars. What does 4 represent? What does x represent?).	
<b>A.CED.3</b> Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context. <i>For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.</i> (A1, M1) <b>a.</b> While functions will often be linear, exponential, or quadratic, the types of problems should draw from more complicated situations. (A2, M3)	<b>A.CED.3a</b> Represent a constraint with an equation or inequality in two variables (e.g., $x + y \leq 8$ , describing the number of boys and girls in an 8-passenger van). (e.g., Abby has \$15 to spend on a snack and two matching T-shirts. If she spends \$3 on the snack, what is the maximum price of each T-	<b>A.CED.3b</b> Create a one-variable constraint using an inequality (e.g., $x \leq 6$ ).	<b>A.CED.3c</b> Demonstrate a constraint using words or models (e.g., how many students can fit at this table?).	



Learning Standard		Complexity a	Complexity b	Complexity c
Most Complex		Least Complex		
		shirt? Key: $x \leq 5$ ).		
<p><b>A.CED.4</b> Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations.</p> <p><b>a.</b> Focus on formulas in which the variable of interest is linear or square. <i>For example, rearrange Ohm's law <math>V = IR</math> to highlight resistance <math>R</math>, or rearrange the formula for the area of a circle <math>A = (\pi)r^2</math> to highlight radius <math>r</math>. (A1)</i></p> <p><b>b.</b> Focus on formulas in which the variable of interest is linear. <i>For example, rearrange Ohm's law <math>V = IR</math> to highlight resistance <math>R</math>. (M1)</i></p> <p><b>c.</b> Focus on formulas in which the variable of interest is linear or square. <i>For example, rearrange the formula for the area of a circle <math>A = (\pi)r^2</math> to highlight radius <math>r</math>. (M2)</i></p> <p><b>d.</b> While functions will often be linear, exponential, or quadratic, the types of problems should draw from more complicated situations. (A2, M3)</p>		<p><b>A.CED.4a</b> Rearrange a one-step formula to highlight a quantity (e.g., use the formula <math>a=lw</math> to highlight the length of the rectangle by rearranging it to <math>l=a/w</math>).</p>	<p><b>A.CED.4b</b> Rearrange a one-step equation to solve for a variable (e.g., solve for <math>x</math>: <math>y = 2 + x</math>, <math>y/2=x</math>).</p>	<p><b>A.CED.4c</b> Match a formula to a given situation (e.g., recognize that <math>a=lw</math> is the formula for the area of a rectangle)</p>
Reasoning with Equations and Inequalities Standards				
Understand solving equations as a process of reasoning and explain the reasoning.				
<p><b>A.REI.1</b> Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.</p>		<p><b>A.REI.1a</b> Order a given sequence of steps to solve an equation (e.g., <math>2x + 5 = 13</math>). Solve two-step equations with integer coefficients and solutions, explaining the steps.</p>	<p><b>A.REI.1b</b> Determine a step needed to solve a two-step equation.</p>	<p><b>A.REI.1c</b> Determine the step needed to solve a one-step equation (e.g., to solve <math>x + 5 = 13</math>, subtract 5 from both sides).</p>
<p><b>A.REI.2</b> Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.</p>		<p><b>A.REI.2a</b> Solve linear equations with more than one step.</p>	<p><b>A.REI.2b</b> Solve 1-step linear equations.</p>	<p><b>A.REI.2c</b> Solve for the missing number within a given number sentence involving addition or subtraction of numbers less than 10.</p>

Learning Standard	Complexity a	Complexity b	Complexity c
Most Complex	Least Complex		
Solve equations and inequalities in one variable.			
<b>A.REI.3</b> Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.	<b>A.REI.3a</b> Solve a two- or three-step linear equation in one variable. Models may be used.	<b>A.REI.3b</b> Solve a one-step linear equation in one variable. Models may be used.	<b>A.REI.3c</b> Given a linear equation in one-variable and a list of possible solutions, identify the solution of the equation.
<b>A.REI.4</b> Solve quadratic equations in one variable. <b>a.</b> Use the method of completing the square to transform any quadratic equation in $x$ into an equation of the form $(x - p)^2 = q$ that has the same solutions. <b>b.</b> Solve quadratic equations as appropriate to the initial form of the equation by inspection, e.g., for $x^2 = 49$ ; taking square roots; completing the square; applying the quadratic formula; or utilizing the Zero-Product Property after factoring.	<b>A.REI.4a</b> Identify or create perfect squares (e.g., square root of $25 = 5$ ).	<b>A.REI.4b</b> Identify equivalent expressions that are cubes (e.g., $m^3 = m \times m \times m$ ).	<b>A.REI.4c</b> Identify equivalent expressions that are squared (e.g., $m^2 = m \times m$ ).
Solve systems of equations.			
<b>A.REI.6</b> Solve systems of linear equations algebraically and graphically. <b>a.</b> Limit to pairs of linear equations in two variables. (A1, M1) <b>b.</b> Extend to include solving systems of linear equations in three variables, but only algebraically. (A2, M3)	<b>A.REI.6a</b> Identify the coordinate at which two lines intersect.	<b>A.REI.6b</b> Locate the point on the graph at which two lines intersect.	<b>A.REI.6c</b> Identify whether two lines intersect.
<b>A.REI.7</b> Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically. <i>For example, find the points of intersection between the line <math>y = -3x</math> and the circle <math>x^2 + y^2 = 3</math>.</i>	<b>A.REI.7a</b> Locate the coordinate of the point(s) at which a line intersects a quadratic function (e.g., at which two coordinates does the line intersect the parabola?).	<b>A.REI.7b</b> Locate the point(s) on the graph at which a line intersects a quadratic function (e.g., identify on the graph where the line intersects the parabola).	<b>A.REI.7c</b> Identify whether a line intersects a quadratic function (e.g., does the line intersect the parabola at one or two points? Does the line intersect the parabola?).

Learning Standard		Complexity a	Complexity b	Complexity c
Most Complex		Least Complex		
Represent and solve equations and inequalities graphically.				
<b>A.REI.10</b> Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).	<b>A.REI.10a</b> Given a graph and an equation, fill out three points on a corresponding table of values.	<b>A.REI.10b</b> Given a table of values, graph the line on the coordinate plane.	<b>A.REI.10</b> Identify a point on a line on a coordinate plane.	
<b>A.REI.11</b> Explain why the x-coordinates of the points where the graphs of the equation $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$ ; find the solutions approximately (e.g., using technology to graph the functions, making tables of values, or finding successive approximations).	<b>A.REI.11a</b> Locate the coordinate at which two lines intersect. Using the x coordinate of the intersection point, substitute it back into the original equation to show that it is a solution of the equation.	<b>A.REI.11b</b> Locate the coordinate point on the graph at which two lines intersect.	<b>A.REI.11c</b> Identify whether two lines intersect.	
<b>A.REI.12</b> Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.	<b>A.REI.12a</b> Given a graph of an inequality including the shaded region, identify three points that make the inequality true.	<b>A.REI.12b</b> Identify on a graph of a line $\leq$ , $\geq$ is represented by a solid line; and $<$ and $>$ are represented by a dotted line.	<b>A.REI.12c</b> Identify the graph of a linear inequality has a shaded region.	




## FUNCTIONS

Learning Standard		Complexity a	Complexity b	Complexity c
Most Complex		Least Complex		
Interpreting Functions Standards				
Understand the concept of a function, and use function notation.				
<b>F.IF.1</b> Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If $f$ is a function and $x$ is an element of its domain, then $f(x)$ denotes the output of $f$ corresponding to the input $x$ . The graph of $f$ is the graph of the equation $y = f(x)$ .	<b>F.IF.1a</b> Determine if a relation is a function.	<b>F.IF.1b</b> Complete an input-output table when given the function rule and values.	<b>F.IF.1c</b> Identify the input or output of a function given in table form.	
<b>F.IF.2</b> Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.	<b>F.IF.2a</b> Given a linear equation using function notation, complete a table of values.	<b>F.IF.2b</b> Represent an equation in $y=$ form with $f(x)$ .	<b>F.IF.2c</b> Understand that $f(x)=y$ .	
<b>F.IF.3</b> Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. <i>For example, the Fibonacci sequence is defined recursively by <math>f(0) = f(1) = 1</math>, <math>f(n + 1) = f(n) + f(n - 1)</math> for <math>n \geq 1</math>.</i>	<b>F.IF.3a</b> Given a sequence, determine the functional rule.	<b>F.IF.3b</b> Predict the next three terms in an arithmetic or geometric sequence (e.g., 3, 6, 9 ...).	<b>F.IF.3c</b> Given a rule, identify the common ratio or common difference in a sequence.	
Interpret functions that arise in applications in terms of the context.				
<b>F.IF.4</b> For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include the following: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity. (A2, M3) <b>a.</b> Focus on linear and exponential functions. (M1) <b>b.</b> Focus on linear, quadratic, and exponential functions. (A1, M2)	<b>F.IF.4a</b> Given a function made up of several linear functions, determine where the function is increasing, decreasing, or flat	<b>F.IF.4b</b> Given a graph of a linear equation, identify the $y$ and/or $x$ intercept.	<b>F.IF.4c</b> Determine whether the linear function is increasing, decreasing, or flat.	

Learning Standard		Complexity a	Complexity b	Complexity c
Most Complex		Least Complex		
<b>F.IF.5</b> Relate the domain of a function to its graph, and, where applicable, to the quantitative relationship it describes. <i>For example, if the function <math>h(n)</math> gives the number of person-hours it takes to assemble <math>n</math> engines in a factory, then the positive integers would be an appropriate domain for the function.</i> <b>a.</b> Focus on linear and exponential functions. (M1) <b>b.</b> Focus on linear, quadratic, and exponential functions. (A1, M2) <b>c.</b> Emphasize the selection of a type of function for a model based on behavior of data and context. (A2, M3)		<b>F.IF.5a</b> Given the graph represented by a linear function, determine the domain.	<b>F.IF.5b</b> Given a context of a linear equation, describe the domain.	<b>F.IF.5c</b> Given a table, state the input values.
<b>F.IF.6</b> Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph. (A2, M3)		<b>F.IF.6a</b> Identify the slope of a line when the equation is written in slope intercept form.	<b>F.IF.6b</b> Identify the slope of a line when given in graph form.	<b>F.IF.6c</b> Determine whether a slope is present on a given visual graph.

Learning Standard	Complexity a	Complexity b	Complexity c
Most Complex	Least Complex		
Analyze functions using different representations.			
<b>F.IF.7</b> Graph functions expressed symbolically and indicate key features of the graph, by hand in simple cases and using technology for more complicated cases. Include applications and how key features relate to characteristics of a situation, making selection of a particular type of function model appropriate. <b>a.</b> Graph linear functions and indicate intercepts. (A1, M1) <b>b.</b> Graph quadratic functions and indicate intercepts, maxima, and minima. (A1, M2) <b>c.</b> Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions. (A2, M3) <b>d.</b> Graph polynomial functions, identifying zeros, when factoring is reasonable, and indicating end behavior. (A2, M3) <b>e.</b> Graph simple exponential functions, indicating intercepts and end behavior. (A1, M1) <b>f.</b> Graph exponential functions, indicating intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude. (A2, M3)	<b>F.IF.7a1</b> Graph a linear function using a graph with a scale of 1. AND/OR <b>F.IF.7a2</b> Determine whether an ordered pair is a viable solution to a given linear function.	<b>F.IF.7b1</b> Determine the y intercept point for a linear graph. AND/OR <b>F.IF.7b2</b> Determine whether the line is increasing (going up), decreasing (going down), or flat.	<b>F.IF.7c1</b> Identify two point on a linear graph. AND/OR <b>F.IF.7c2</b> Classify graphs of functions as linear or non-linear.

Learning Standard	Complexity a	Complexity b	Complexity c
Most Complex			
Analyze functions using different representations.			
<p><b>F.IF.8</b> Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.</p> <p><b>a.</b> Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context. (A2, M3)</p> <p><b>i.</b> Focus on completing the square to quadratic functions with the leading coefficient of 1. (A1, M2)</p> <p><b>b.</b> Use the properties of exponents to interpret expressions for exponential functions. <i>For example, identify percent rate of change<sup>6</sup> in functions such as <math>y = (1.02)^t</math>, and <math>y = (0.97)^t</math> and classify them as representing exponential growth or decay.</i> (A2, M3)</p> <p><b>i.</b> Focus on exponential functions evaluated at integer inputs. (A1, M2)</p>	<p><b>F.IF.8a</b> Identify equivalent expressions (e.g. <math>2x + 2x = 4x</math>, <math>x^2 * x^2 = x^4</math>).</p>	<p><b>F.IF.8b</b> Identify equivalent expressions (limit to three terms) (e.g. <math>x + x + x = 3x</math>, <math>x * x * x = x^3</math>).</p>	<p><b>F.IF.8c</b> Identify equivalent expressions (limit to two terms) (e.g. <math>x + x = 2x</math>).</p>
<p><b>F.IF.9</b> Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). <i>For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.</i> (A2, M3)</p> <p><b>a.</b> Focus on linear and exponential functions. (M1)</p> <p><b>b.</b> Focus on linear, quadratic, and exponential functions. (A1, M2)</p>	<p><b>F.IF.9a</b> Compare a function given in table form to another function given in graphical form. <i>For example, which one is increasing?</i></p>	<p><b>F.IF.9b</b> Match a function given in table form to its graph.</p>	<p><b>F.IF.9c</b> Match a function given as a verbal description to its graph. <i>For example, which is an increasing function?</i></p>

Most Complex		Least Complex	
Build a function that models a relationship between two quantities.			
	F.BF.1a Create a linear function that represents a linear relationship between quantities in a given context.	F.BF.1b Given a linear function that describes a real-world situation and given the value of one variable, find and interpret the value of the other variable.	F.BF.1c Identify the meaning of each number and/or variable in a linear function that describes a real-world situation.
	F.BF.2a Identify the rule for a pattern.	F.BF.2b Identify the next term in a pattern.	F.BF.2c Determine if a given set represents a pattern.
Build new functions from existing functions.			
F.BF.3 Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$ , $kf(x)$ , $f(kx)$ , and $f(x + k)$ for specific values of $k$ (both positive and negative); find the value of $k$ given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them. (A2, M3) a. Focus on transformations of graphs of quadratic functions, except for $f(kx)$ . (A1, M2)	F.BF.3a Identify a line reflected over the y-axis on a coordinate grid.	F.BF.3b Identify a line reflected over the x-axis on a coordinate grid.	F.BF.3c Identify a line on a coordinate grid.

Learning Standard		Complexity a	Complexity b	Complexity c
Most Complex		Least Complex		
<b>F.BF.4</b> Find inverse functions. <b>a.</b> Informally determine the input of a function when the output is known. (A1, M1)		<b>F.BF.4a</b> Solve for $x$ when $y$ is given (e.g. $y = x + 3$ ; what is the value of $x$ when $y$ is 5?).	<b>F.BF.4b</b> Identify the input and output of a function.	<b>F.BF.4c</b> Identify a function.
<b>Linear Quadratic and Exponential Models Standards</b>				
<i>Construct and compare linear, quadratic, and exponential models, and solve problems.</i>				
<b>F.LE.1</b> Distinguish between situations that can be modeled with linear functions and with exponential functions. <b>a.</b> Show that linear functions grow by equal differences over equal intervals and that exponential functions grow by equal factors over equal intervals. <b>b.</b> Recognize situations in which one quantity changes at a constant rate per unit interval relative to another. <b>c.</b> Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.		<b>F.LE.1a</b> Identify a situation that represents a linear and/or exponential function.	<b>F.LE.1b</b> Identify the graph of a linear function and an exponential function.	<b>F.LE.1c</b> Identify a graph of a linear function.
		<b>F.LE.2a</b> After creating a sequence, make a graph that represents that sequence.	<b>F.LE.2b</b> Create a geometric sequence of at least 5 numbers with models.	<b>F.LE.2c</b> Create an arithmetic sequence with a model.
		<b>F.LE.3a</b> Observe a situation that shows increasing and decreasing linear and exponential events. (e.g., doubling a penny every day gives you more money than receiving \$100 a day for a month)	<b>F.LE.3b</b> Identify an exponential function.	<b>F.LE.3c</b> Identify if a linear function is increasing or decreasing.
<b>F.LE.2</b> Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (including reading these from a table).				
<b>F.LE.3</b> Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly or quadratically. (A1, M2)				



Learning Standard		Complexity a	Complexity b	Complexity c
Most Complex		Least Complex		
F.I.E.4 For exponential models, express as a logarithm the solution to $ab^{ct}=d$ where $a$ , $c$ , and $d$ are numbers, and the base $b$ is 2, 10, or $e$ ; evaluate the logarithm using technology.	F.I.E.4a Identify equivalent expressions with exponents.	F.I.E.4b Identify equivalent expressions with exponents (limit to power 3 expressions) (e.g. Which is the same as $m \times m \times m$ ?).	F.I.E.4c Identify equivalent expressions with exponents (limit to power 2 expressions) (e.g. Which is the same as $m \times m$ ?).	
	Interpret expressions for functions in terms of the situation they model.			
F.I.E.5 Interpret the parameters in a linear or exponential function in terms of a context.	F.I.E.5a Given a context, interpret the parameters of an exponential function (e.g., during $x$ weeks, the existing number of fish in the pond has been doubled). This situation is modeled by the exponential function $y = 100(2^x)$ , where 100 is the initial number of fish in the pond, 2 is a growth factor, $(2^x)$ is the number by which the initial number of fish, 100, is multiplied for every increase in $x$ , and $y$ is the total number of fish in the pond.	F.I.E.5b Given a context, interpret the parameters of a linear function (e.g., Marsha has \$10 already saved and saves an additional \$5 a week for $x$ number of weeks). This situation is modeled by a linear function $f(x) = 5x + 10$ , where 10 is the initial amount that has been saved, 5 is the weekly saving, $5x$ is the amount of money saved during $x$ weeks, and $f(x)$ is a total amount of money saved including the initial amount.	F.I.E.5c Identify the constant in a linear or exponential equation.  OR When given the graph of a function, identify the $y$ intercept.	
Trigonometric Functions Standards				
Extend the domain of trigonometric functions using the unit circle.				
F.TF.1 Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.	F.TF.1a Identify the measure of a central angle on a circle when the measure of the arc is given.	F.TF.1b Identify the measure of an angle.	F.TF.1c Identify an angle.	

Learning Standard		Complexity a	Complexity b	Complexity c
Most Complex		Least Complex		
F.TF.2 Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.		F.TF.2a Identify the measure of a central angle on a circle when the measure of the arc is given.	F.TF.2b Identify the measure of an angle.	F.TF.2c Identify an angle.
	Model periodic phenomena with trigonometric functions.			
	F.TF.5 Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.	F.TF.5a Identify the measure of a central angle on a circle when the measure of the arc is given.	F.TF.5b Identify the measure of an angle.	F.TF.5c Identify an angle.
Prove and apply trigonometric identities.				
F.TF.8 Prove the Pythagorean identity $\sin^2(\theta) + \cos^2(\theta) = 1$ , and use it to find $\sin(\theta)$ , $\cos(\theta)$ , or $\tan(\theta)$ given $\sin(\theta)$ , $\cos(\theta)$ , or $\tan(\theta)$ and the quadrant of the angle.		F.TF.8a Find the hypotenuse when the length of the sides is given.	F.TF.8b Identify the parts of a right triangle.	F.TF.8c Identify a right triangle.

**GEOMETRY**

Learning Standard	Complexity a	Complexity b	Complexity c
Most Complex	Least Complex		
Congruence			
Experiment with transformations in the plane.			
<b>G.CO.1</b> Know precise definitions of ray, angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and arc length.	<b>G.CO.1a</b> Identify points, lines, line segments, angles (right, acute, obtuse, and order by size), and perpendicular and parallel lines.	<b>G.CO.1b</b> Identify points, lines, line segments and angles (right, acute, obtuse, and order by size).	<b>G.CO.1c</b> Identify points, lines, and line segments, and order angles by size.
<b>G.CO.2</b> Represent transformations in the plane using, e.g., transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not, e.g., translation versus horizontal stretch.	<b>G.CO.2a</b> Demonstrate that a rotation (turn), a reflection (flip), or a translation (slide) maps a figure onto another.	<b>G.CO.2b</b> Identify whether a rotation (turn), a reflection (flip), or a translation (slide) can map a figure onto another.	<b>G.CO.2c</b> Match shapes in different orientations. (i.e., shapes = 2D)
<b>G.CO.3</b> Identify the symmetries of a figure, which are the rotations and reflections that carry it onto itself. <b>a.</b> Identify figures that have line symmetry; draw and use lines of symmetry to analyze properties of shapes. <b>b.</b> Identify figures that have rotational symmetry; determine the angle of rotation, and use rotational symmetry to analyze properties of shapes.	<b>G.CO.3a</b> Show that two figures have symmetry on a coordinate plane.	<b>G.CO.3b</b> Identify figures that have line symmetry or rotational symmetry, using concrete objects or on a coordinate plane.	<b>G.CO.3c</b> Given visual models, determine which figures have line symmetry. (i.e., figure =3D)
<b>G.CO.4</b> Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments.	<b>G.CO.4a</b> Identify that a translation requires a direction and distance; a rotation requires a center and angle; and a reflection requires a line.	<b>G.CO.4b</b> Identify whether a transformed figure is a "translation," "reflection," or "rotation."	<b>G.CO.4c</b> Identify whether a transformed figure is a "slide," "flip," or "turn."

Learning Standard		Complexity a	Complexity b	Complexity c
Most Complex		Least Complex		
G.CO.5 Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using items such as graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another.	G.CO.5a Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure.	G.CO.5b Given visuals or real-world items, demonstrate a rotation (turn), a reflection (flip), or a translation (slide).	G.CO.5c Match shapes in different orientations.	
	Understand congruence in terms of rigid motions.			
	G.CO.6 Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent	G.CO.6a Identify a basic rigid motion (a rotation (turn), a reflection (flip), or a translation (slide) that maps one figure onto another. (Restrict to situations in which a single basic rigid motion suffices.)	G.CO.6b Show two figures are congruent by demonstrating that a rotation (turn), a reflection (flip), or a translation (slide) maps one onto the other.	G.CO.6c Match shapes to show congruence by placing one figure on top of the other.
G.CO.7 Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent.	G.CO.7a Identify whether a rotation (turn), a reflection (flip), or a translation (slide) is required to show that a triangle is congruent to another triangle on a coordinate plane. Limit to one transformation.	G.CO.7b Identify whether a rotation (turn), a reflection (flip), or a translation (slide) is required to show that a triangle is congruent to another triangle. Limit to one transformation.	G.CO.7c Match triangles in different orientations.	
G.CO.8 Explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow from the definition of congruence in terms of rigid motions.	G.CO.8a Determine whether two triangles are congruent using ASA, SAS, or SSS.	G.CO.8b Match corresponding parts (sides and angles) of congruent triangles.	G.CO.8c Match one corresponding part (side or angle) of two congruent triangles.	