Montessori Lesson Plan

School: Craig Montessori School	Teacher: Shamika Johnson
Subject/Topic(s): Math	
Grade Level: 8	Theme: Community
Curriculum Components Included: ☐Project ☐Mini-Whole Grp ☐Lesson-Small Grp ☐Student engagement during lesson ☐Shelfwork ☐Rubric ☐Self-Assessment ☐Seminar/Qs ☐Interdisciplinary ☐ Outside Opportunity	
Seven Gateways for Adolescence addressed in this lesson: ☐Deep Connection ☐Silence & Solitude ☐Meaning & Purpose ☐Joy & Delight ☐Creative ☐Transcendence ☐Initiation	

Standards/Objectives

8.G.B.6: Explain a proof of the Pythagorean Theorem and its converse.

8.G.B.7: Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions.

Student Friendly Language:

8.G.B.6

I can use algebraic reasoning to relate a visual model to the Pythagorean Theorem.

8.G.B.7

I can draw a diagram and use the Pythagorean Theorem to solve real world problems involving right triangles.

Math Practice 6: Attend to precision.

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

Materials: Teacher	Materials: Student	Time/Dates
- Pythagorean Theorem Material	- Notebook - Pencil	- October·

Facts/Skills (Prior Knowledge)	Concepts/Big Ideas
- Students understand squares/square numbers - Students know the parts of a triangle	- Essential Question: How can you test the converse of the pythagorean theorem and use it to solve problems? - Understanding that the 2 squares on the legs of a right triangle make up the area of the square on the hypotenuse.

Lesson Relates to Theme (Note: Every content lesson will not directly relate to the theme)

- N/A

Connection to Elementary Material or Lesson

The use of the Pythagorean Theorem material

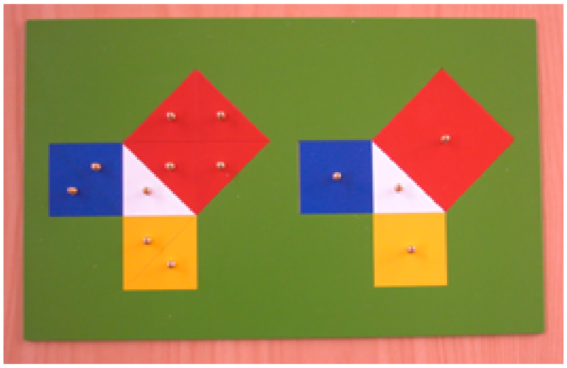
Step-by-Step Procedures	
1st Period Lesson – 20 minutes (Include steps and materials) - Open - Invite students to the lesson table with the Pythagorean Montessori material and present the lesson - See below for the presentation of the lesson	
2nd Period – Recognition (Shelfwork) - Work with the Pythagorean plates - Work with the Paper version of the plates .	2nd Period – Recall Practice
3rd Period – Student Application - Solve Pythagorean proofs	
Plan for Differentiation	

<table>
<tr><th><u>Teaching</u></th><th><u>Work</u></th><th><u>Assessment</u></th></tr>
<tr>
<td>- Based on the students understanding I will either present them with plates I or plates II</td>
<td>Floaters
- Work through three part cards and the level I plates to understand the proof
Swimmers
- Students will practice using the level II plates and practice the 3 part cards
Divers
- Students get a lesson on the Euclidean proof for the Pythagorean theorem and work through and understand that proof</td>
<td>- The puzzle that they receive will be reflective of the Material that they used.</td>
</tr>
<tr><td colspan="3"><u>Outside Support: Who, What, How</u>
SpEd Teacher - Per each student’s IEP</td></tr>
<tr><td colspan="3">Formal Assessments</td></tr>
<tr><td colspan="3"><u>Formative Assessments</u>
- The students will receive a “puzzle” and they must solve the puzzles to show their understanding of the Pythagorean Theorem and then answer questions A-C</td></tr>
<tr><td colspan="3"><u>Summative Assessment</u>
N/A</td></tr>
</table>

Equivalence with Pythagorean Insets

Remember we have already mentioned Pythagoras' name when we talked about the history of geometry, the rope. We have also explored right-angled triangles with the box of sticks: right isosceles with the neutral sticks and right scalene with special 3-4-5 combinations.

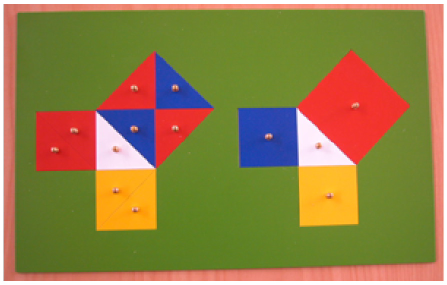
Presentation "Pythagorean Plate I or Sensorial Plate"



1. Ask children to identify the white triangle: …right isosceles. Identify other shapes … squares.
2. We are going to discuss something about the relationship between these two (yellow and blue) whole squares and this whole one (red).
3. The sides of these two squares are the same as the legs of the triangle.
4. The length of the side of the large square is the same as the hypotenuse.
5. Demonstrate equivalences sensorial.
   Red triangles to red square and replace.
   Yellow triangles to yellow square and replace.
   Blue triangles to blue square and replace

May have to review "leg"

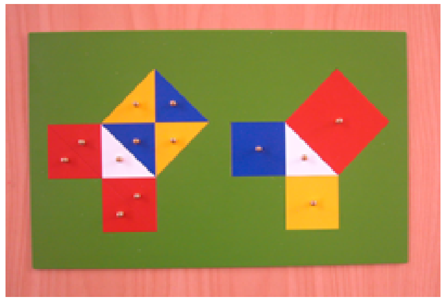
6. Now let's look at these divided squares … something interesting!
   Remove two red triangles – interchange with two blue triangles.



7. Remove other two red triangles –

interchange with yellow triangles

Now what can we say about this red square?
It is equal to the blue plus the yellow!
Also, the blue equals half the red and the yellow equals half the red.



Extension:

Children can trace in their notebooks

After some time of independent work try to draw out the Pythagorean Theorem:

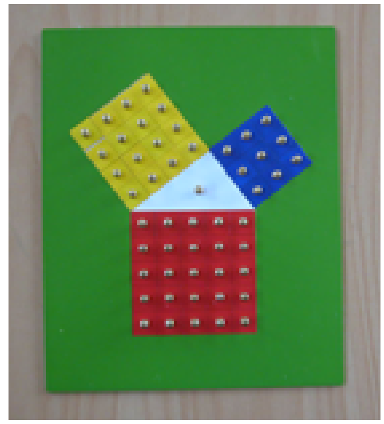
In a right triangle (remove the white triangle) the sum of the squares built on the legs is equal to the square constructed on the hypotenuse.

Explain to the children that this is the same as the early rope experiences of the Egyptians.

This is an early experience because it is demonstrated sensorially.

Presentation "Plate II – Numerical"

The material: triangle 3cm leg – 9
4cm leg – 16
5cm leg - 25



1. Here we have another demonstration of the theorem of Pythagoras. In this inset all the squares are divided up so that we can use numbers of squares.
2. We are going to try to prove that this square plus this one equals this one… that the squares built on the legs equal to the square built on the hypotenuse.
3. The children interchange pieces.
4. Show the numerical value: $3^2 = 9$
   $4^2 = 16$
   $5^2 = 25$

Children may discover the proportions of the triangle. In that case you might introduce the idea of the Pythagorean triples. This is also a good time for work problems. For example:
Given: length of two legs
Find: the hypotenuse

Given: length of leg and hypotenuse
Find: length of other leg.

If children can do this second example you know you have finished the work because they have internalized the process.

It is your job to make sure that the children are ready to discard the material and work abstractly.

Pythagorean Theorem with Constructive Triangles

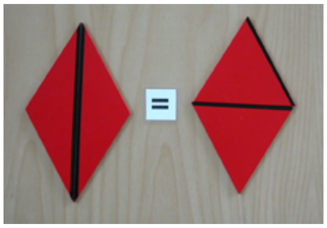
Presentation… with plane figures other than squares

Material: Constructive Triangles: hexagonal boxes H1 and H2, and triangle box T

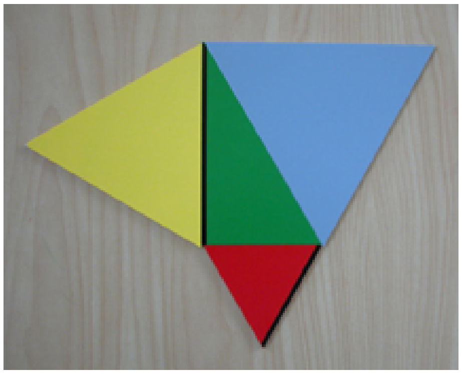
1. Remove green right scalene from triangle box. Identify: leg, leg, hypotenuse.
2. What do we already know about the relationship between the length of the hypotenuse and the length of the legs. "The sum of the squares is equal to the length of the hypotenuse" SHOW PYTHAGOREAN TEMPLATE 1
3. This theorem is usually expressed in terms of squares. Have you ever wondered since the square is a regular polygon, if there is also was a relationship with other regular polygons built on the legs of a right angle triangle?

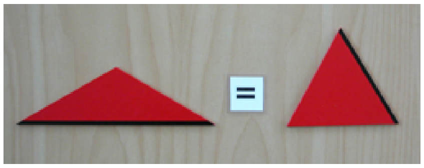
   Place gray equilateral (T box) on hypotenuse
   Small red equilateral on shorter leg
   Yellow equilateral (H1 box) on longer leg
4. We already know something about the relationship between these equilateral triangles.

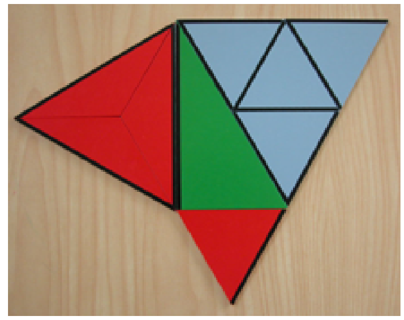
   Demonstrate equivalences you may have done before:



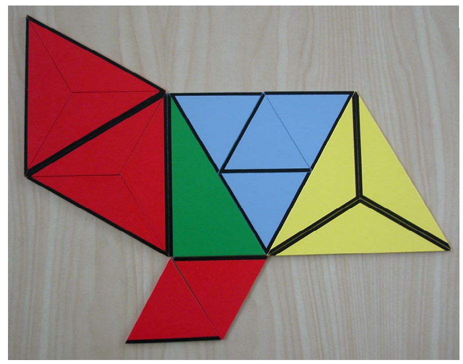
The square built on the legs is equal to the square built on the hypotenuse.



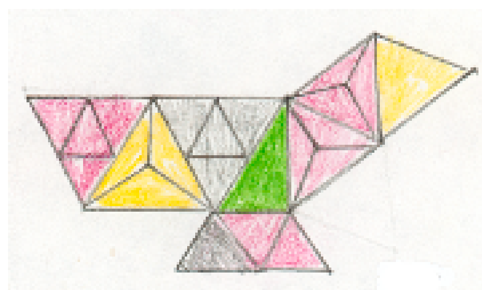




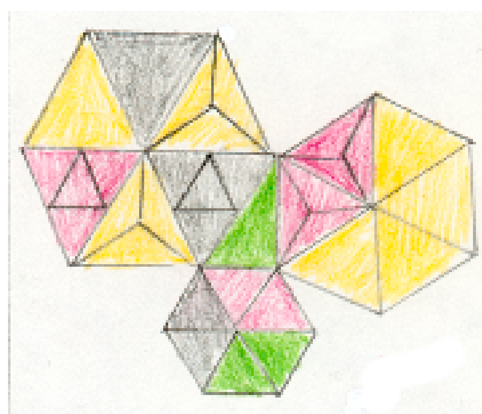
5. Exchange four gray equilaterals for the large one.
   Exchange three red obtuse isosceles for the yellow equilateral.
6. All these little triangles are equivalent.
7. Remember we said that maybe these would equal this one? Let's see… Here we have one and three, which equals four, and here we also have four!!



8. So it also works for equilateral triangles.
9. I wonder what else we could make. What if we doubled the equilateral triangles? Do it! We get another series of regular polygons … rhombi
   2 + 6 = 8



10. What if we added a third equilateral triangle? (add wholes) … trapezoids
    3 + 9 =12



11. Doubling this (with paper yellow equilaterals) makes hexagons:
    6 + 18 = 24
12. Another way to look at these is to treat the large gray equilateral triangle as the unit of measure.

    Then in the first case we would have:
    1/4 + 3/4 = 4/4

In the case of the rhombi:
2/4 + 6/4 = 8/4

The trapezoids:
3/4 + 9/4 = 12/4

The hexagons:
(6 x 1/4) + (18 x 1/4) = 24/4

Pythagoras Three (Euclidean Logic)

The Pythagoras three plate

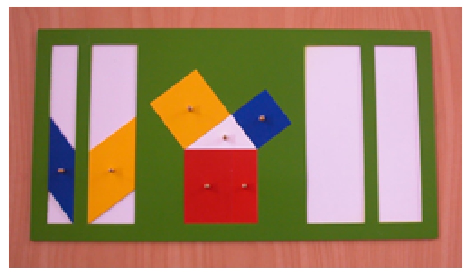
<u>Note</u>:

The children have already worked with the Pythagorean theorem, which states that the sum of the squares built on the legs of a right triangle equal the square built on the hypotenuse.

Presentation

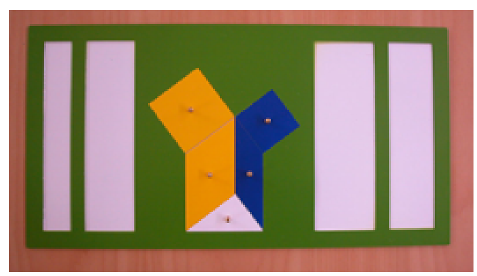
1. Introduce the plate. Do you remember the Pythagorean theorem? "Yes: The sum of the squares built on the legs of a right angled triangle equals the square built on the hypotenuse." We are going to use this plate and try to prove that these red rectangles equal the blue square plus the yellow square.

1



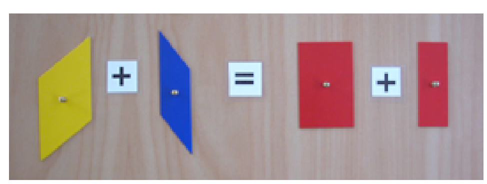
2. Remove the red rectangles. Slide the white triangle down and place the yellow and blue parallelograms in the space. Replace pieces as in 1.

2



3. The sum of the blue and yellow parallelograms is equivalent to the sum of the two red rectangles – (that form the square built on the hypotenuse). Return pieces as in 1.

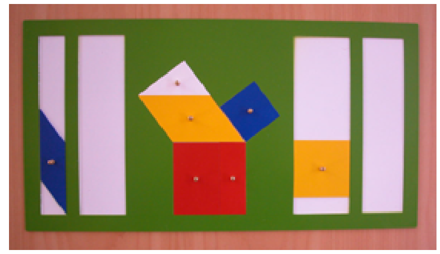
3



4. Remove the yellow square and slide white triangle up as shown in (4). Replace space with yellow parallelogram. Does this

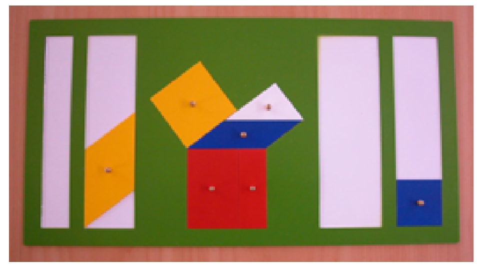
4

demonstrate that the yellow parallelogram is equivalent to the yellow square? Return pieces as in 1.



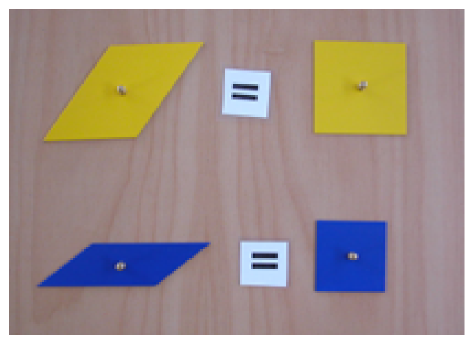
5. Remove the blue square and slide the triangle up as shown in 5. Replace space with blue parallelogram. Does this demonstrate that the blue parallelogram is equivalent to the blue square? Return pieces as in 1.

5

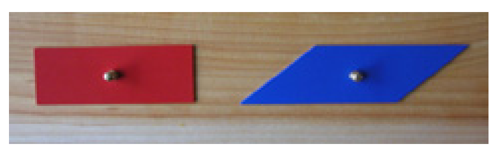


6. Therefore, we can see that the yellow parallelogram is equal to the yellow square and the blue paralegal is equal to the blue square. (6).

6

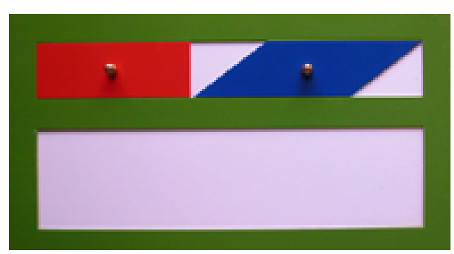


7. Take the small red rectangle and the blue parallelogram. Considering the longer sides as base, identify the altitude and base of each figure… sensorially show that they are the same.



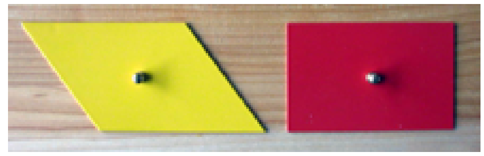
8. These two figures are equivalent because b=b and h=h.

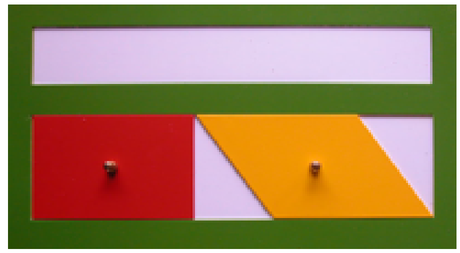
   Now take the small red rectangle and place in the frame (You can turn plate vertically) as shown. Note that the altitudes are equal and that the length of the whole opening will hold both pieces perfectly!



9. Take large red rectangle and yellow parallelogram. Demonstrate that b=b and h=h and are therefore equivalent to each other!

   Place large red rectangle in frame as shown we did in (1) above. Note that the altitudes are equal and that the length of the whole opening will hold both pieces perfectly!





10. ANOTHER WAY.
    Take blue parallelogram and place in frame as shown; also place yellow parallelogram in frame as shown. Note that the bases are the short sides!

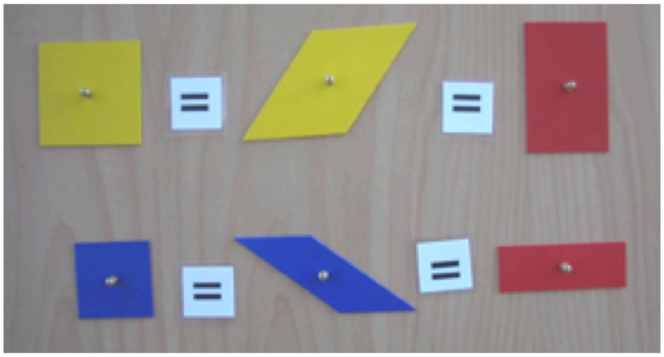
    Show that the blue square is equivalent to the blue parallelogram because b=b and h=h.

    Similarly show equivalence of yellow square and yellow parallelogram.

11. We have shown that
    small red rectangle = blue parallelogram
    and
    blue parallelogram = blue square

12. We have also shown that
    large red rectangle = yellow parallelogram
    and
    yellow parallelogram = yellow square

13. Therefore:



This could not be shown directly because their measurements are not commensurable… cannot be measured with same unit!

The study of areas and metric system will fall between these two ages. Also in that time frame will be the arithmetic demonstration of the extensions of Pythagorean Theorem will also be after the areas.

<u>Summarize:</u> We have show that the square built on the shorter leg is equivalent to the smaller rectangle, which makes up the square of the hypotenuse; the square built on the longer leg makes up the larger rectangle, which forms part of the square of the hypotenuse. The sum of the squares of the legs is equal to the square of the hypotenuse.

<u>Ages</u>

For presentation of first two Pythagorean Frames
8 1/2 +

For third (Euclidean) Frame   11 1/2

Problem 3.2

Copy the shapes on the previous page or use the puzzle pieces your teacher gives you.

A Study a triangle piece and the three square puzzle pieces. How do the side lengths of the squares compare to the side lengths of the triangle?

B
1. Arrange the 11 puzzle pieces to fit exactly into the two puzzle frames.
2. What conclusion can you draw about the relationship among the areas of the three colored squares?
3. What does the conclusion you reached in part (2) mean in terms of the side lengths of the triangles?
4. Compare your results with those of another group. Did that group come to the same conclusion your group did? Is this conclusion true for all right triangles? Explain.

C Suppose a right triangle has legs of length 3 centimeters and 5 centimeters.
1. Use your conclusion from Question B to find the area of a square drawn on the hypotenuse of the triangle.
2. What is the length of the hypotenuse?

D In Problem 3.1 and Problem 3.2, you have explored the Pythagorean Theorem, a relationship among the side lengths of a right triangle. State this theorem as a rule for any right triangle with leg lengths a and b and hypotenuse length c.

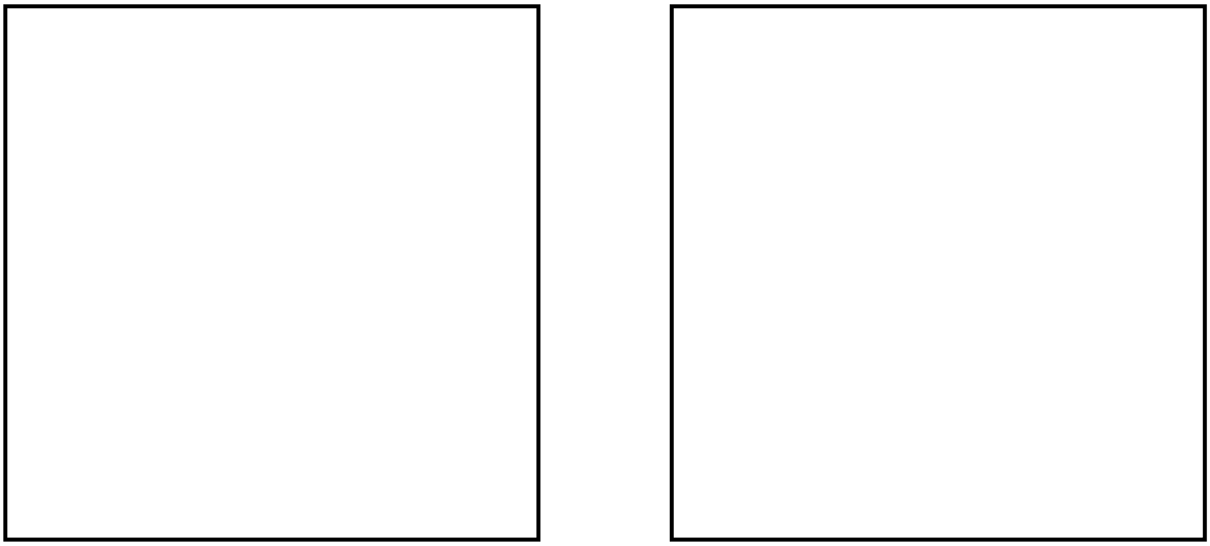
ACE Homework starts on page 49.

Name .... Date .... Class ....

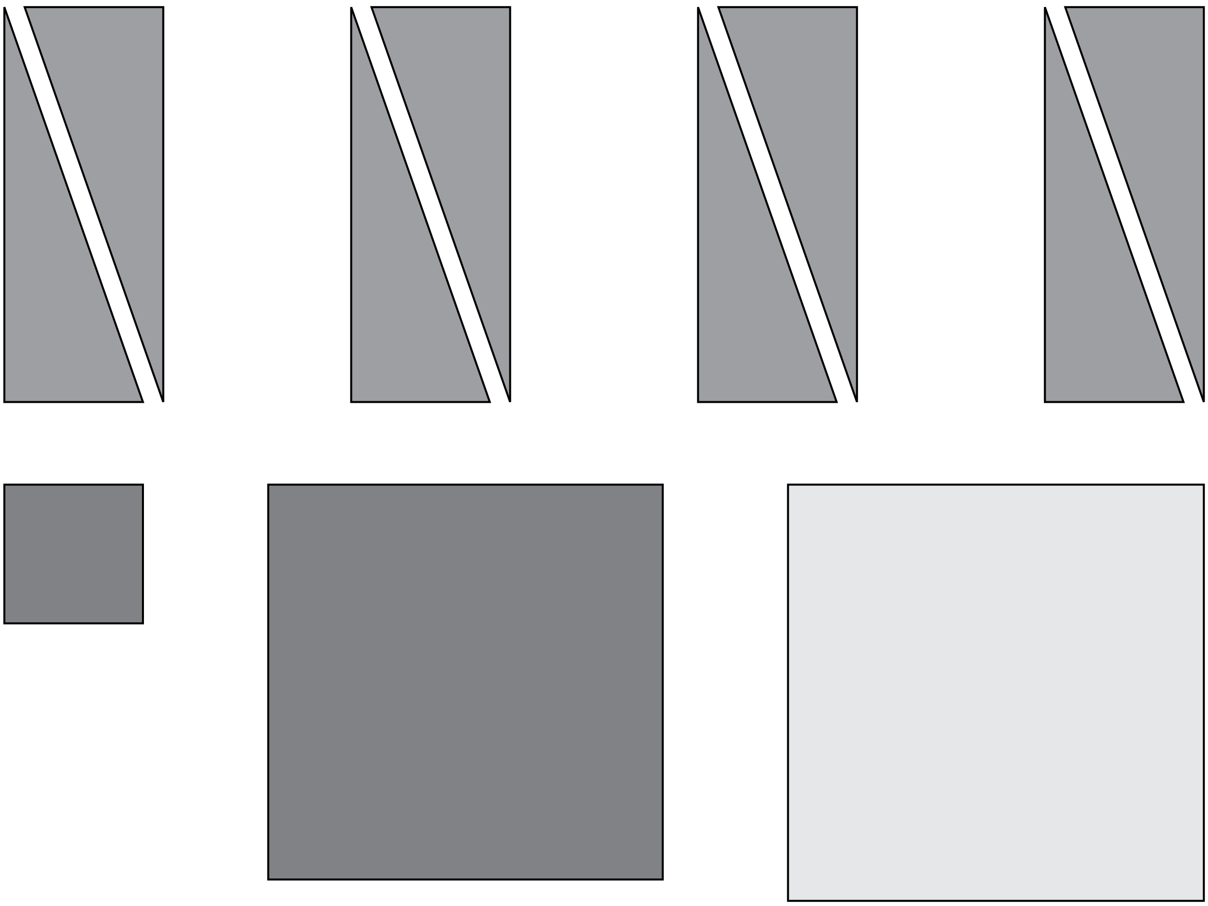
Labsheet 3.2A

Puzzle Set A

Frames



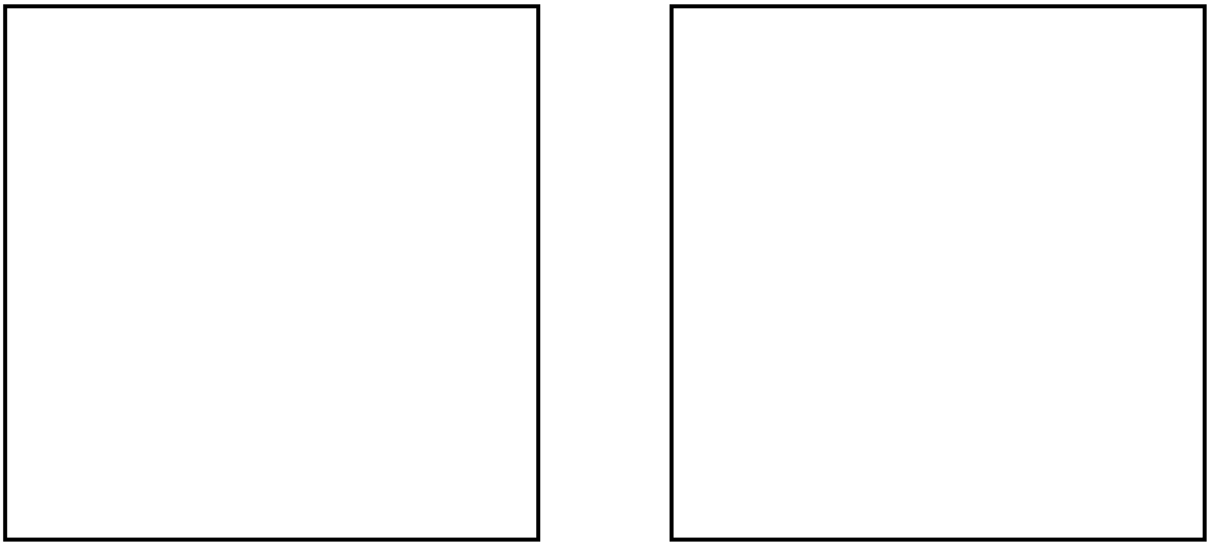
Puzzle Pieces



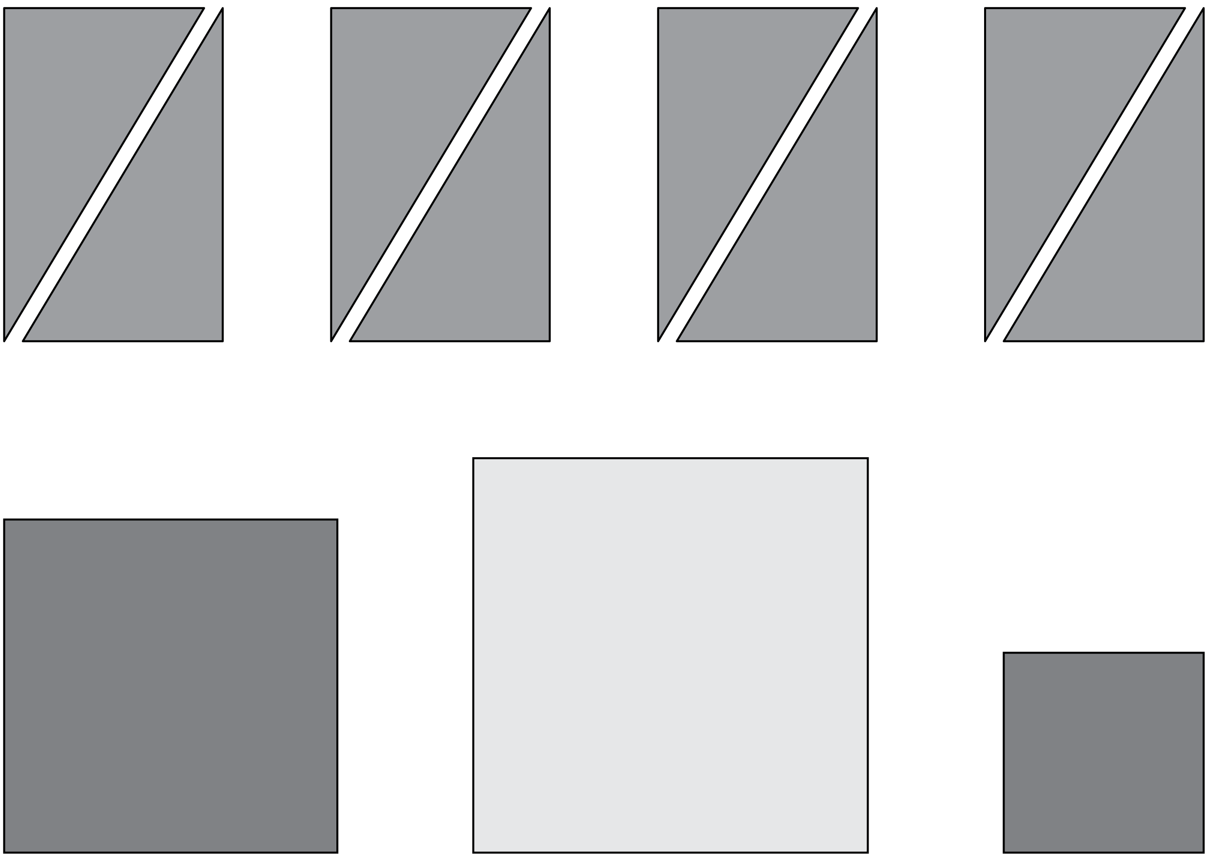
Labsheet 3.2B

Puzzle Set B

Frames



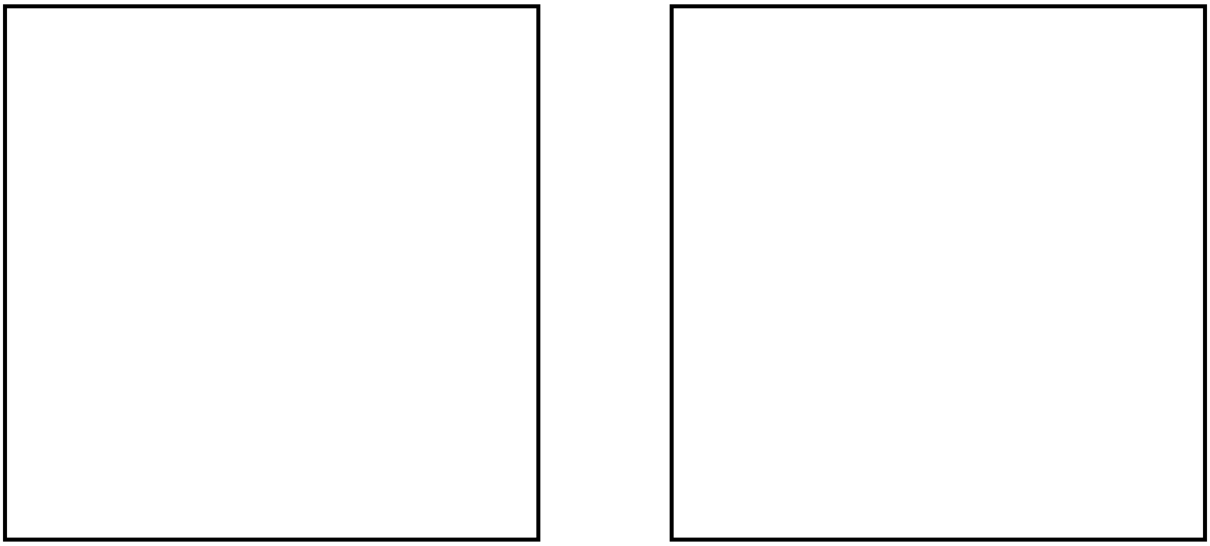
Puzzle Pieces



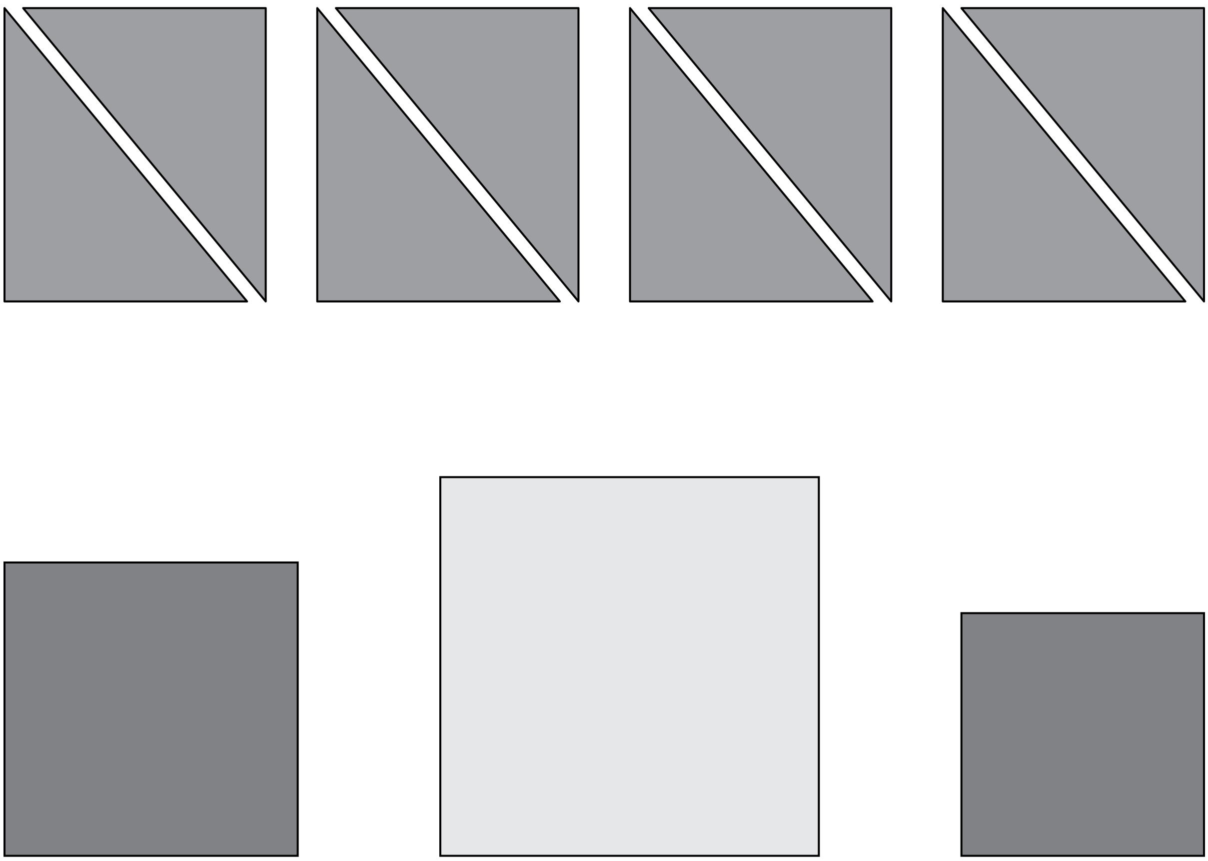
Labsheet 3.2C

Puzzle Set C

Frames

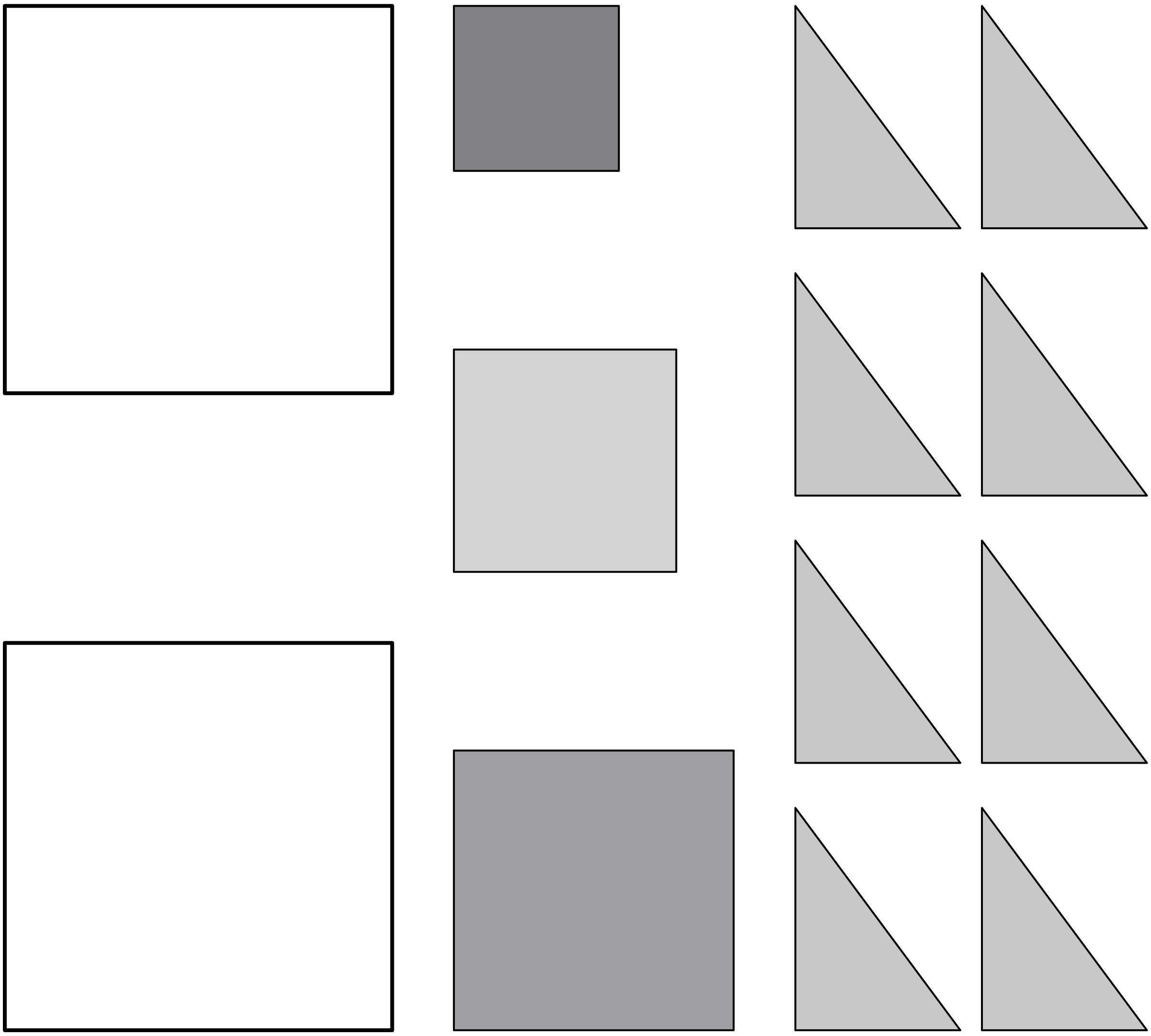


Puzzle Pieces



Labsheet 3.2D

Puzzle Set D



Frames

Puzzle Pieces